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Tilt angle optimization to maximize incident solar radiation: A review

Amit Kumar Yadav, S.S. Chandel *

Centre for Energy and Environment, National Institute of Technology, Hamirpur, Himachal Pradesh 177005, India

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ABSTRACT

The tilt angle of a solar energy system is one of the important parameters for capturing maximum solar radiation falling on the solar panels. This angle is site specific as it depends on the daily, monthly and yearly path of the sun. The accurate determination of the optimum tilt angle for the location of interest is essential for maximum energy production by the system. A number of methods have been used for determining the tilt angle at different locations worldwide. Keeping in view the relevance of the optimum tilt angle in energy production and reducing the cost of solar energy systems, the present study has been undertaken. This paper provides the update status of research and applications of various methods for determining solar panel tilt angle using different optimization techniques. The study shows that for maximum energy gain, the optimum tilt angle for solar systems must be determined accurately for each location. The review will be useful for designers and researchers to select suitable methodology for determining optimal tilt angle for solar systems at any site.

Contents

1. Introduction

The design and development of solar energy systems require the knowledge of variation and maximum utilization of solar radiation falling on it. The measured solar radiation data are not available for large number of sites, so it has to be estimated. A number of techniques are used for the estimation of solar radiation on horizontal surface [1–3]. The amount of solar radiation received by a solar photovoltaic [PV] panel or a solar thermal collector is mainly affected by its orientation and tilt angle [4,5]. The solar panels are generally oriented toward the equator, in the northern hemisphere oriented toward south and in the southern hemisphere toward north. However, the solar radiation is site specific with diurnal, monthly, seasonal and yearly variations; as such the optimum tilt angle for capturing maximum solar radiation will also vary for every location. This study provides an update status of research and applications of various computational

* Corresponding author. Tel.: +91 1972254748; fax: +91 1972223834.
E-mail addresses: sschandel2013@gmail.com, chandel_shyam@yahoo.com (S.S. Chandel).

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Nomenclature

β_{opt}	optimum tilt angle
GA	genetic algorithm
$\overline{H}$	monthly average daily total radiation on horizontal surface
H_B	daily beam radiation incident on an inclined surface, $Wh/m^2/day$
$\overline{H}_b$	monthly average daily beam radiation
H_D	daily sky-diffuse radiation incident on an inclined surface, $Wh/m^2/day$
H_d	daily diffuse radiation incident on a horizontal surface, $Wh/m^2/day$
$\overline{H}_d$	monthly average daily diffuse radiation
H_g	daily global radiation incident on a horizontal surface, $Wh/m^2/day$
H_R	daily ground reflected radiation incident on an inclined surface, $Wh/m^2/day$
H_s	sky diffuse radiation
H_T	daily global radiation on a tilted surface, $Wh/m^2/day$
$\overline{H}_T$	monthly average daily radiation on tilted surface
I	total irradiation on horizontal surface (J/m^2)
I_d	diffuse radiation on horizontal surface (J/m^2)
$I_{d,T}$	diffuse radiation on tilted surface
I_{ext}	extra-terrestrial daily radiation incident on a horizontal surface, $Wh/m^2/day$
I_{sc}	solar constant 1367 W/m^2
I_T	total radiation on tilted surface
n	nth day of the year
PSO	Particle Swarm Optimization
PV	photo voltaic
R_b	ratio of average daily beam radiation incident on an inclined surface to that on a horizontal surface
R	ratio of total radiation on tilted surface to total radiation on horizontal surface
$\overline{R}$	ratio of monthly average daily radiation on tilted surface to monthly average daily total radiation on horizontal surface ($\overline{H}_T/\overline{H}$)
$\overline{R}_b$	ratio of average daily beam radiation on tilted surface to horizontal surface for a month
R_d	ratio of average daily diffuse radiation incident on an inclined surface to that on a horizontal surface
w_{sr}	sunrise hour angle (°)
w_{ss}	sunset hour angle (°)
β	surface slope from the horizontal°
δ	declination angle (°)
ϕ	latitude (°)
ρ	ground albedo
γ	surface azimuth angle (°)

methods for determining solar panel tilt angle using different optimization techniques. This paper is organized as follows: Section 2 describes the methods to determine solar radiation on sloped surfaces. Section 3 provides details of optimum tilt angle determination. An overview of optimum tilt angle determination methods is given in Section 4. The conclusions of the study are given in Section 5.

2. Solar radiation modeling on sloped surfaces

In this section, basic solar radiation parameters and models to determine solar radiation on sloped surfaces are described. The solar radiation incident outside the earth's atmosphere, called extraterrestrial radiation for a day of the year, n is given as

$$I_{ext} = \left\{ \begin{array}{l} I_{sc}[1.0+0.333\cos(360n/365)] \\ I_{sc}[1.000110+0.034211\cos((n-1)(360/365))+0.00128\sin((n-1)(360/365))+0.000719\cos 2((n-1)(360/365))+0.000077\sin 2((n-1)(360/365)) \end{array} \right\} \quad (1)$$

2.1. Solar time

The solar time is given as follows:

$$\text{Solar time} = \text{standard time} + 4(L_{st} - L_{loc)+E} \quad (2)$$

$$E = 229.2(0.000075+0.001868\cos B - 0.032077\sin B - 0.014615\cos 2B - 0.04089\sin 2B) \quad (3)$$

where $B=(n-1)(360/365)$, L_{st} is the standard meridian for the local time zone, L_{loc} is the longitude of the location.

2.2. Solar geometry

The position of the sun in the sky is given by zenith angle (θ_z) (Fig. 1) [6]. The angular position of the sun at solar noon is called declination (δ); $-23.45° \le \delta \le 23.45°$. The angle between the plane of the surface and horizontal is called slope (β); $0° \le \beta \le 180°$. The deviation of the projection on a horizontal plane of the normal to the surface from the local meridian, with zero due south, east negative and west positive, is called surface azimuth angle (γ); $-180° \le \gamma \le 180°$. The hour angle ($w$) is the angular displacement of the sun east or west of the local meridian due to the rotation of the earth on its axis at 15° per hour with morning negative and afternoon positive.

2.3. Global, beam, diffuse irradiance and the sky radiance distribution

The distribution of solar radiance over the sky (Fig. 2) consists of three components: isotropic dome, circumsolar and horizon brightening [7]. The circumsolar and horizon brightening are assumed to be concentrated at the center of the sun and at the horizon. The horizon brightening for clear skies is the highest at the horizon and its intensity decreases away from the horizon and for overcast skies. The sky radiance increases instead of decreasing away from the horizon. The anisotropic models include diffuse sky radiation in the circumsolar and horizontal brightening components of the solar radiation. The isotropic models assume that the intensity of diffuse sky radiation is uniform over the sky dome.

2.4. Solar radiation on sloped surfaces

The determination of solar radiation on sloped surface requires the values of total solar radiation on the horizontal surface and the direction from which beam and diffuse radiation component reaches the surface. The distribution of diffuse radiation over the sky dome is affected by variable cloudiness and atmospheric clarity conditions. The diffuse solar radiation model is composed

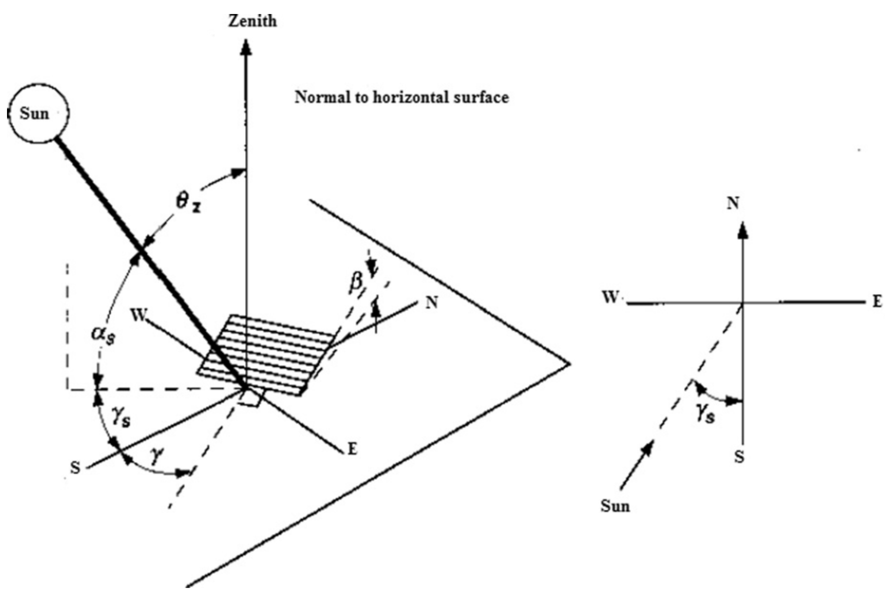

Fig. 1. Zenith angle, slope, surface azimuth angle and solar azimuth angle for tilted surface.

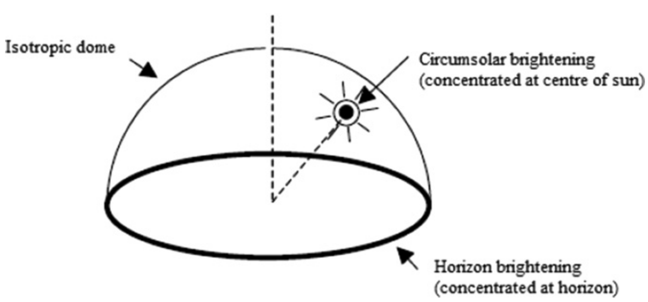

Fig. 2. Schematic view of the sky showing solar radiance distribution component.

of three parts: isotropic part which receives solar radiation uniformly from the entire sky dome; in circumsolar part, diffusion takes place due to forward scattering of solar radiation; horizon brightening takes place in clear skies and near the horizon. Therefore, the angular distribution of diffuse radiation is a function of reflectance (albedo) of ground. A high albedo value results in reflection of solar radiation back to the sky resulting in horizon brightening. Thus, the solar radiation incident on tilted surface (I_T) is given by the relation:

$$I_T = I_{T,b} + I_{T,d,iso} + I_{T,d,cs} + I_{T,d,hz} + I_{T,refl} \quad (4)$$

where $I_{T,b}$ is the beam radiation, $I_{T,d,iso}$ is the isotropic component, $I_{T,d,cs}$ is the circumsolar component, $I_{T,d,hz}$ is the horizon brightening component of diffuse radiation and $I_{T,refl}$ is reflected radiation. When radiation view factor is taken into consideration, (I_T) is as follows:

$$I_T = I_b R_b + I_{d,iso} F_{c-s} + I_{d,cs} R_b + I_{d,hz} F_{c-hz} + I\rho F_{c-g} \quad (5)$$

Pandey and Katiyar [8] investigated different diffuse components of solar radiation at different tilt angles (15°, 30°, 45° and 60°) at Lucknow (Latitude 26.75°, Longitude 80.85°), India. The Circumsolar, Isotropic model, Klucher model and Hay's anisotropic model were used to calculate monthly mean daily diffuse radiation on the inclined surface. The comparison of measured and estimated monthly mean daily diffuse radiation on inclined surface shows that the Klucher model provides a good estimation of the sky diffuse radiation on inclined surface for northern Indian region conditions with mean percentage errors of less than 7%. David et al. [9] used four (Hay, Skartveit and Olseth, Gueymard, Perez) models to estimate diffuse radiation on tilted surface for Reunion Island. The Perez model exhibits best performance and albedo constant i.e. 0.2 provides accurate results. Bortolini et al. [10] developed a correlation of diffuse fraction in terms of clearness index to estimate diffuse solar radiation for European sites. The error in correlations models is found to be less than 20%, showing accurate estimation of diffuse radiation.

A number of models have been proposed by different researchers [11–25] to calculate total solar radiation on tilted surfaces from global radiation on horizontal surface. Temps and Coulson [19] introduced geometrical terms into the isotropic model to take into account the brightening of the sky in the region of the sun and at the horizon. The only difference among these models is in the assessment of the sky-diffuse component. Based on the assumptions made, the estimation models can be classified into isotropic [12] and anisotropic [13,15] ones.

2.4.1. *Isotropic model*

This model approximates the condition of an overcast sky and assumes that the intensity of sky diffuse radiation is uniform over the sky dome. Liu and Jordan [12] modified Hottel and Woertz's model [11] by including beam, isotropic diffuse and solar radiation diffusely reflected from the ground and obtained the equation for I_T as follows:

$$I_T = I_b R_b + I_d \left(\frac{1+\cos\beta}{2}\right) + I\rho\left(\frac{1-\cos\beta}{2}\right) \quad (6)$$

The isotropic sky model for monthly mean solar radiation on unshaded tilted surface is considered [18,19] and $\overline{H}_T$ is given as follows:

$$\overline{H}_T = \overline{H}_b \overline{R}_b + \overline{H}_d \left(\frac{1+\cos\beta}{2}\right) + \overline{H}\rho\left(\frac{1-\cos\beta}{2}\right) \quad (7)$$

For the surface in the northern hemisphere sloped toward the equator, i.e. for surfaces with $\gamma = 0°$ the equation for $\overline{R}_b$ is given as below.

$$\overline{R}_b = \frac{\cos(\phi-\beta)\cos\delta\sin w_s' + (\pi/180) w_s' \sin(\phi-\beta)\sin\delta}{\cos\phi\cos\delta\sin w_s + (\pi/180) w_s \sin\phi\sin\delta} \quad (8)$$

where w_s' is the sunset hour angle for the tilted surface for the

mean day of the month and is given by

$$w_s' = \min\begin{bmatrix}\cos^{-1}(-\tan\phi\tan\delta)\\ \cos^{-1}(-\tan(\phi-\beta)\tan\delta)\end{bmatrix} \quad (9)$$

For surfaces in the southern hemisphere sloped toward the equator, with $\gamma = 180^\circ$ $\overline{R}_b$ is given as follows.

$$\overline{R}_b = \frac{\cos(\phi+\beta)\cos\delta\sin w_s' + (\pi/180)w_s'\sin(\phi+\beta)\sin\delta}{\cos\phi\cos\delta\sin w_s + (\pi/180)w_s\sin\phi\sin\delta} \quad (10)$$

$$w_s' = \min\begin{bmatrix}\cos^{-1}(-\tan\phi\tan\delta)\\ \cos^{-1}(-\tan(\phi+\beta)\tan\delta)\end{bmatrix} \quad (11)$$

where declination angle $\delta = (23.45\pi/180)\sin(2\pi(284+n/365))$.

The isotropic diffuse model presents a good fit to solar radiation data under overcast skies and tends to underestimate I_T under clear and partly cloudy conditions. Klein and Theilacker (KT method) [25] assumed both diffuse and ground reflected radiation as isotropic to calculate average radiation on sloped surfaces. This method showed improved results over the isotropic method when compared with hourly calculation of radiation data.

$$\overline{R} = \frac{\cos(\phi-\beta)}{d\cos\phi}\left[\left(a - \frac{\overline{H}_d}{\overline{H}}\right)\left(\sin w_s' - \frac{\pi w_s'}{180}\cos w_s''\right) + \frac{b}{2}\left(\frac{\pi w_s'}{180} + \sin w_s'(\cos w_s' - 2\cos w_s'')\right)\right] \quad (12)$$

where $w_s'' = \cos^{-1}(-\tan(\phi \pm \beta)\tan\delta)$, the positive sign is used in the northern hemisphere for south-facing surfaces and the negative sign for the southern hemisphere for north-facing surfaces $d = \sin w_s - (\pi w_s/180)\cos w_s$, $a = 0.409 + 0.5016\sin(w_s - 60)$, $b = 0.6609 - 0.4767\sin(w_s - 60)$.

The developed equations in this method are valid for any azimuth angle for all latitudes. If $\gamma \neq 0^\circ$ or 180° the times of sunrise and sunset will not be symmetrical at solar noon; $\overline{R}_b$ is formulated as follows and it is valid in case when the sun rises or sets on the surface twice each day:

$$\overline{R} = D + \frac{\overline{H}_d}{\overline{H}}\left(\frac{1+\cos\beta}{2}\right) + \rho\left(\frac{1-\cos\beta}{2}\right) \quad (13)$$

$$D = \begin{bmatrix}\max(0, G(w_{ss}, w_{sr})) & \text{if } w_{ss} \geq w_{sr}\\ \max(0, (G(w_{ss}, -w_{sr}) + G(w_{ss}, w_{sr}))) & \text{if } w_{ss} > w_{sr}\end{bmatrix} \quad (14)$$

$$|w_{sr}| = \min\left[w_s, \cos^{-1}\frac{AB + C\sqrt{A^2 - B^2 + C^2}}{\sqrt{A^2 + B^2}}\right] \quad (15)$$

$$w_{sr} = \left[-|w_{sr}| \quad \text{if}(A > 0 \text{ and } B > 0) \text{ or } (A \geq B) \left|w_{sr}\right| \quad \text{otherwise}\right] \quad (16)$$

$$|w_{ss}| = \min\left[w_s, \cos^{-1}\frac{AB - C\sqrt{A^2 - B^2 + C^2}}{\sqrt{A^2 + B^2}}\right] \quad (17)$$

$$w_{ss} = \left[|w_{ss}| \text{ if}(A > 0 \text{ and } B > 0) \text{ or } (A \geq B) \quad -|w_{ss}| \quad \text{otherwise}\right] \quad (18)$$

where $A = \cos\beta + \tan\phi\cos\gamma\sin\beta$, $B = \cos w_s\cos\beta + \tan\delta\sin\beta\cos\gamma$, $C = (\sin\beta\sin\gamma/\cos\phi)$.

This method is useful for surfaces with surface azimuth angles more than 15° from the south.

2.4.2. Anisotropic model

Klucher [15] modified the Temps and Coulson model [19] by incorporating conditions of cloudy skies given by the following relation [26]:

$$H_s = \frac{1}{2}H_d(1+\cos\beta)\left[1 + F\sin^3(\beta/2)\right](1 + F\cos^2\theta\sin^3\theta_z) \quad (19)$$

where F is a modulating function given by $F = 1-(H_d/H)^2$. When the sky is overcast, the value of F is zero and relation (19) becomes isotropic. When value of F tends to one, Eq. (19) can be used for the clear sky conditions as given by Temps and Coulson [19]. Hay's anisotropic model [27] consists of an isotropic as well as circumsolar component. However, horizon brightening as incorporated by Klucher [15] is not taken into consideration. Therefore, Hay presented the following relation for the daily sky diffuse radiation on tilted surface:

$$H_s = H_d\left[\frac{H - H_d}{I_{ext}}\overline{R}_b + \frac{1}{2}(1+\cos\beta)\left(1 - \frac{H - H_d}{I_{ext}}\right)\right] \quad (20)$$

Temps and Coulson [19] incorporated horizon brightening on clear days by using correction factor $(1+\sin^3(\beta/2))$ to the isotropic diffuse model. By adding modulation factor (f) Reindl et al. [14] modify the Hay and Davies model and predicted $I_{d,T}$ as follows:

$$I_{d,T} = I_d\left[(1-A_i)\left(\frac{1+\cos\beta}{2}\right) + \left(1 + f\sin^3\left(\frac{\beta}{2}\right)\right) + A_iR_b\right] \quad (21)$$

where $f = \sqrt{I_b/I}$ and A_i is an anisotropic index as a function of atmosphere transmittance for beam radiation. The HDKR (Hay, Davies, Klucher, Reindel) model considered beam and ground reflected terms and predicted as follows:

$$I_T = (I_{b+I_d}A_i)R_b + I_d(1-A_i)\left(\frac{1+\cos\beta}{2}\right)\left(1 + f\sin^3\left(\frac{\beta}{2}\right)\right) + I\rho\left(\frac{1-\cos\beta}{2}\right) \quad (22)$$

This model gives total radiation on tilted surface toward the equator and gives results close to the measured values.

Perez et al.'s model [20] considered all the components of diffuse radiation and $(I_{d,T})$ is as follows.

$$I_{d,T} = I_d\left[(1-F_1)\left(\frac{1+\cos\beta}{2}\right) + F_1\frac{a}{b} + F_2\sin\beta\right] \quad (23)$$

where $a = \max(0, \cos\theta)$; $b = \max(\cos 85, \cos\theta_z)$ accounts for the angle of incidence of the cone of circumsolar radiation; brightness coefficients are given by $F_1 = \max[0, (f_{11} + f_{12}\Delta + (\pi\theta_z/180)f_{13})]$; $F_2 = (f_{21} + f_{22}\Delta + (\pi\theta_z/180)f_{23})$; clearness $\varepsilon = ((I_{d+I_{b,n}})/I_d + 5.535 \times 10^{-6}\theta_z^3)/\ (1 + 5.535 \times 10^{-6}\theta_z^3)$; brightness $\Delta = m(I_d/I_{on})$ where m and I_{on} are the air mass and incidence beam radiation respectively. Therefore, by adding beam, diffuse from isotropic, circumsolar, horizon and ground reflected radiation I_T is given by

$$I_T = I_bR_b + I_d(1-F_1)\left(\frac{1+\cos\beta}{2}\right) + I_{dF_1}\frac{a}{b} + I_dF_2\sin\beta + I\rho\left(\frac{1-\cos\beta}{2}\right) \quad (24)$$

This model gives slightly higher value of I_T in comparison to the measured values. This model is recommended for surfaces with γ far from 0° in the northern hemisphere or 180° in the southern hemisphere.

3. Optimum tilt angle determination

The optimum tilt angle determination of a solar panel is fundamental to its efficient operation because incorrect positioning leads to loss of potential solar power. The optimum tilt angle calculations are based on maximizing the solar radiation falling on a sloped surface using different optimization techniques.

3.1. Total solar radiation on tilted surface

The monthly average daily total solar radiation on a tilted surface (H_T) is the sum of the direct or beam solar radiation (H_B), diffuse radiation (H_D), and ground reflected radiation (H_R) on the tilted surface. Thus, for a surface tilted at an angle from the

horizontal, the incident total radiation is given by the relation:

$$H_T = H_B + H_D + H_R \tag{25}$$

The daily beam radiation received on the tilted surface may be expressed as

$$H_B = (H_g - H_d)\overline{R}_b \tag{26}$$

where H_g and H_d are the monthly mean daily global and diffuse radiations on a horizontal surface. The $\overline{R}_b$ for the surface in the northern and southern hemispheres sloped toward the equator are given by relations (8) and (10).

The methods to estimate the ratio of diffuse solar radiation on a tilted surface to that of a horizontal are classified as the isotropic and anisotropic models. The sky-diffuse radiation can be expressed as

$$H_D = R_d H_d, \tag{27}$$

where R_d is given by different authors as shown in Table 1 [26].

As a consequence, the total solar radiation on a tilted surface can thus be expressed as

$$H_T = (H_g - H_d)\overline{R}_b + H_g \rho \frac{1-\cos\beta}{2} + H_d R_d \tag{28}$$

Therefore, total solar radiation falling on tilted surface is computed by varying tilt angle (β) from 0° to 90°. The optimum tilt angle is taken at which solar radiation on the tilted surface H_T becomes maximum.

3.2. Tilt angle–latitude relations

A number of authors have determined the optimum angle–latitude (ϕ) relations, which are generally used for determining the optimum tilt angles used as thumb rules by solar energy system installers for many locations (Table 2) [42]. These methods give simple and approximate values for tilt angle; however, the accurate optimum angle for a particular site should be determined using the measured values of solar radiation for the site.

The latitude based optimum tilt angle relations do not take into account the factors that affect solar radiation like altitude above the sea level and cloud coverage patterns, so these methods do not give accurate results for the cloudy regions like Northern Europe. It is difficult to determine their accuracy for different sites, but these relations are mostly used by PV industry for solar system installation. The NREL's PV Watts calculator—the free online tool, [41], can also be used with site specific meteorological data to find solar energy yield at any tilt and the optimum tilt is taken at which solar energy yield is maximum. Charles Landau recommended that the tilt angles in winter seasons are about 5° steeper than the commonly recommended tilt angle [41]. In winter season, the solar radiation is mostly incident at midday so panel should be mounted at solar noon to collect maximum solar energy throughout the day. In summer season, the sun moves lower in the sky in morning and evening so it is necessary to tilt less to the south (or more to the north) to collect maximum solar radiation.

The calculation of tilt angle by maximizing solar radiation on sloped surfaces can generally be done for sites lying between 5°N

Table 1
Values of parameter R_d for different sky diffuse models.

Model	Authors	Mathematical relations for R_d
Isotropic	Badescu [28]	$R_d = \frac{3+\cos(2\beta)}{4}$
Isotropic	Tian et al. [29]	$R_d = 1 - \frac{\beta}{180}$
Isotropic	Koronakis [30]	$R_d = \frac{1}{3}[2+\cos\beta]$
Isotropic	Liu and Jordan [24]	$R_d = \frac{1+\cos\beta}{2}$
Anisotropic	Reindl et al. [14]	$R_d = \frac{H_b}{H_o}R_b + \left(1-\frac{H_b}{H_o}\right)\left(\frac{1+\cos\beta}{2}\right)\left(\left(1+\sqrt{\frac{H_b}{H_g}}\sin^3\left(\frac{\beta}{2}\right)\right)\right.$
Anisotropic	Skarveit and Olseth [17]	$R_d = 0.51R_b + \frac{1+\cos\beta}{2} - \frac{1.74}{1.26\pi}\left[\sin\beta - \left(\frac{\beta\pi}{180}\right)\cos\beta - \pi\sin^3\left(\frac{\beta}{2}\right)\right]$ where $\Omega = \left\{\max\left[0, \left(0.3 - 2\frac{H_b}{H_o}\right)\right]\right\}$
Anisotropic	Steven and Unsworth [31]	$R_d = 0.51R_b + \frac{1+\cos\beta}{2} - \frac{1.74}{1.26\pi}\left[\sin\beta - \left(\frac{\beta\pi}{180}\right)\cos\beta - \pi\sin^3\left(\frac{\beta}{2}\right)\right]$
Anisotropic	Hay [27]	$R_d = \frac{H_b}{H_o}R_b + \left(1-\frac{H_b}{H_o}\right)\left(\frac{1+\cos\beta}{2}\right)$
Anisotropic	Klucher [15]	$R_d = \frac{1}{2}(1+\cos\beta)\left[1+F\sin^3(\beta/2)\right](1+F\cos^2\theta\sin^3\theta_z)$ where $F = 1-(I_d/I)^2, \cos\theta = \sin\delta\sin(\phi-\beta)+\cos\delta\cos(\phi-\beta)\cos w$, $\sin\theta_z = \sqrt{1-(\sin\delta\sin\phi+\cos\delta\cos\phi\cos w)^2}, w = \cos^{-1}(-\tan\phi\tan\delta)$

Table 2
Optimum tilt angle–latitude angle relations.

References	Recommended tilt angle	Remarks
Duffie and Beckman [6]	$(\phi+15°)\pm15°$	These methods are suitable for calculating approximate tilt angles for different latitudes. The minus sign is used for summer season and the positive sign for winter season.
Heywood [32]	$\phi-10°$	
Lunde [33]	$\phi\pm15°$	
Chinnery [34]	$\phi+10°$	
Löf and Tybout [35]	$\phi+(10°\rightarrow30°)$	
Garg [36]	$\phi+15°$, $\phi-15°$, 0.9ϕ	
Hottel [37]	$\phi+20°$	
Kern and Harris [38]	$\phi+10°$	
Yellot [39]	$\phi+20°$	
Elminir et al. [40]	$(\phi+15°)\pm15°$	

and 40° N latitudes with sunny climates where beam radiation component dominates. The areas located beyond 45°N, like countries in Northern Europe namely Ireland, England, Norway and Sweden, receive maximum amount of diffuse radiation due to frequent heavy cloud cover. Therefore, the methodologies of choosing tilt angle by maximizing the beam radiation are unsuitable, as other atmospheric conditions are not considered.

Armstrong and Hurley [42] developed a methodology to determine optimum tilt angle for locations with frequently overcast skies using monthly sunshine duration data and hourly cloud observations. Under cloudy skies, it is important to differentiate between direct and diffuse radiation for a particular site to calculate optimum tilt angle so the Perez model is useful to calculate diffuse radiation falling on the solar panel. The tilt angle is changed from 0° to 90° in steps of 1° and is selected as the one that maximizes the incident solar radiation on solar panel by taking into account the frequencies of cloudy skies. The optimum tilt angle of grid connected and standalone PV system is selected that matches the available solar radiation with the load demand.

3.3. Tilt angle optimization using GA, SA and PSO techniques

Optimization is a method of finding the conditions that give maximum or minimum value of the objective function to obtain the accurate results under specified conditions. It is useful in the design, construction and maintenance of any engineering system [43]. For finding optimum tilt angle, solar radiation on the tilted surface (Eq. (28)) is taken as an objective function which is solved using different optimization techniques like Genetic Algorithm (GA), Simulated Annealing (SA) technique and particle swarm optimization (PSO).

Genetic algorithm (GA) is suitable for optimization problems which include complex nonlinear variables, as it can find the global optimum solution with a high probability [44,45]. A population of points is used for starting the GA instead of a single design point [46]. It involves principles of natural genetics and natural selection. The natural genetics are reproduction, crossover and mutation which are used in the genetic search procedure [47]. Talebizadeha et al. [48] used GA to calculate hourly, daily, monthly, seasonally and yearly optimum tilt angle for Iran and showed that the optimum hourly surface azimuth angle is not zero and optimum tilt angles of photovoltaic panels and solar collector are found to be the same. The solar energy gain at daily, monthly optimum tilt angle is found to be the same but energy gain is more at hourly tilt angle. Therefore, the solar tracker is beneficial for hourly variation of tilt angle. Čongradac et al. [49] used GA and fuzzy logic process to find out optimum blind tilt angle (angle rotated in anticlockwise and clockwise direction) for maintaining accurate brightness of the room. This process is useful in maintaining user's comfort and saving energy.

Simulated Annealing (SA) derives its name from the simulation of thermal annealing of critically heated solids and is used to find the global optimum with a high probability of objective functions which contain numerous local minima. Chen et al. [50] used SA for calculating the optimum installation angle for fixed solar-cell panels.

Particle Swarm Optimization (PSO) is a stochastic technique for exploring the search space for optimization [51]. It is achieved by particles in multidimensional space that have a position and a velocity [52]. In PSO particles are flying through hyperspace and adjust their own best position i.e. the smallest objective function values. Chang [53] used the varying inertia weight methods [54–58] and proposed a particle-swarm optimization method with nonlinear time-varying evolution (PSO—NTVE) to determine the tilt angle of PV modules for maximum output electrical energy of the modules in Taiwan. The yearly optimal angle for Taipei area is 18.16° and 17.3°, 16.15°, 15.79°, 15.17°, 17.16°, 15.94° for Taichung, Tainan, Kaosiung, Hengchung, Hualian, Taitung respectively. The PSO—NTVE is faster than the other three PSO methods and the GA method for achieving an optimum solution. Therefore, these optimization techniques will give better results than the other methods (latitude based method, maximizing radiation by changing β from 0° to 90° at fixed steps).

3.4. Tilt angle optimization using ANN techniques

Artificial Neural Network techniques [ANN] are excellent tools for research as these are able to solve non-linear function approximation, data classification, clustering and simulation [59]. These are used in different fields of science and technology [60–63] and also for the solar radiation predictions [64]. Chang [65] used sequential neural-network approximation and orthogonal arrays (SNAOA) to determine the optimum tilt angle for maximum output power energy of PV modules in seven areas of Taiwan. The back propagation neural network is used to search optimum tilt angle design. The maximum and minimum values of optimum tilt angle and voltage are used as constraint to maximize power on PV module. The annual optimal angle is found to be 23.25° for Taipei site. Mehleri et al. [66] used Radial Basis Function neural network (RBFNN) to predict solar radiation on tilted surface using tilt angle and orientation as input parameters. Fuzzy algorithm is used for training RBFNN and it is found that changing tilt angle seasonally improves performance and uniformity of PV array output power. Mehleri et al. [67] incorporates global solar irradiance on a horizontal surface, extraterrestrial radiation, solar zenith angle and solar incidence angle on a tilted plane as inputs to RBFNN for estimating global solar irradiance on inclined surfaces in Athens. The absolute fraction of variance (R^2) is 96%, proving the estimation by the model accurate. Notton et al. [68] developed three ANN models for Mediterranean site of Ajaccio, France, to estimate hourly global irradiation on the tilted plane. The first and second ANN models estimate hourly global irradiation on 45°, 60° tilted planes respectively whereas the third ANN model estimates both on 45°, 60° tilted planes. The ANN models incorporate declination angle, hour, zenith angle, hourly extraterrestrial horizontal irradiation, hourly global irradiation as inputs and hourly global irradiation on 45°, 60° tilted planes as output parameters. The (R^2) in the first, second, third ANN models are 99.79%, 99.82% and 99.70% respectively. Celik and Muneer [69] used generalized regression neural networks (GRNN) to predict solar radiation on the tilted surface for Iskenderun, Turkey. The GRNN utilize input parameters as global solar irradiation on horizontal surface, declination and hour angles. The R^2 and mean absolue percentage error are 98.7% and 14.9 Wh/m^2 respectively. Chatterjee and Keyhani [70] used 14 inputs (latitude, ground reflectivity and 12 months irradiance value) to estimate optimum tilt and total irradiance on tilted surface by ANN. The activation function in hidden, output layers is hyperbolic tangents, and linear and Levenberg–Marquardt algorithm is used for training. The difference between predicted and analytically determined optimum tilt angle is found to be 3° showing accurate estimation by the ANN techniques Therefore, the ANN techniques reduce the computation part of optimum tilt angle calculations, and estimate optimum angle and tilted surface solar radiation at different sites with better accuracy.

4. Overview of optimum tilt angle determination methods

Many authors have determined optimum tilt angles analytically and experimentally by searching for the maximum total solar radiation falling on the PV modules for a number of locations. Some of the relevant highlights are given in this section. Asl-Soleimani et al. [71] determined optimum tilt angle for Tehran using an experimental setup of five 11-Wp PV mono-crystalline modules by measuring the maximum energy output for a period of

Table 3
Summary of optimum tilt angle correlations.

Authors	Description
Elsayed [83]	The correlation of optimum tilt angle β_{opt} in terms of clearness index (K_T), latitude (ϕ), day number (D) is given by: $\beta_{opt} = (6-4.8K_T + 0.86K_T^{0.27}\phi + 0.0021\phi^2) + (31K_T^{0.37} + 0.094K_T^{0.46}\phi + 0.00063K_T^{-0.17}\phi^2)\cos\left[\frac{360}{365}(D+11.5)\right]$
Tiris and Tiris [84]	The optimum tilt angle (β_{opt}) as a function of clearness index (K_T), declination angle (δ) for Gebze in Turkey are as follows, $\beta_{opt}=33.24-1.31\delta$; $\beta_{opt}=22.09+25.79K_T^2-1.49\delta$; $\beta_{opt}=35.15-1.37\delta-0.007\delta^2$; $\beta_{opt}=35.15-1.39\delta-0.007\delta^2-4.26\times10^{-5}\delta^3$. The variation of root mean square error (RMSE) is from 1.09 to 1.74 and R^2 from 99.40 to 99.77.
Elkassaby [85]	The monthly β_{opt} in terms of latitude (ϕ), Julian Day (M) is given by (i) From January to March ($1\le M\le 3$) $\beta_{opt}=60.00012+1.49986M+3.49996M^2+(\phi-30)\times(0.7901+0.01794M+0.0165M^2)$. (ii) From April to June ($3\le M\le 6$) $\beta_{opt}=216.0786-72.03219M+6.00312M^2+(\phi-40)\times(1.07515+0.11244M-0.03749M^2)$. (iii) From July to September ($6\le M\le 9$) $\beta_{opt}=29.118311-20.52981M+2.50186M^2+(\phi-50)\times(-11.17256+2.70569M-0.15035M^2)$. (iv) From October to December ($9\le M\le 12$) $\beta_{opt}=-441.2385+84.54332M-3.50196M^2+(\phi-40)\times(4.2137-0.54834M+0.0223M^2)$. Providing that if $\beta_{opt}<0$ then $\beta_{opt}=0$. The design of solar collector at negative tilt angle is significant between latitude 0° and ±25°.
Ertekin et al. [86]	The monthly optimum tilt angles of the south facing collectors as a sinusoidal function of latitude (ϕ), and day of the year D over Turkey is given by $\beta_{opt}=25.521438+26.838291\cos(-0.017844\phi+1.013901D+7.527742)$. The R^2, RMSE of correlation equation are 98.8% and 2.06% respectively.
Skeiker [87]	The daily ($\beta_{opt,d}$), monthly ($\beta_{opt,m}$) optimum tilt angle for Syria are as follows: $\beta_{opt,d}=\phi-\tan^{-1}\left[\frac{w_{ss}}{\sin(w_{ss})}\tan(\delta)\right]$ $\beta_{opt,m}=\phi-\tan^{-1}\left(\frac{\sum_{n=n_1}^{n=n_2}((24/\pi)I_{sc}[1+0.034\cos(2\pi n/365)\sin(\delta)w_{ss})}{\sum_{n=n_1}^{n=n_2}((24/\pi)I_{sc}[1+0.034\cos(2\pi n/365)\cos(\delta)\sin(w_{ss}))}\right)$ where n_1, n_2 are the first, last days of the mth month as counted from January. The yearly optimum tilt angle is 30.56° and energy gains are 28%, 26% at monthly, season tilt angle.
Moghadam et al. [88]	The annual optimum $\beta_{opt}=0.197+0.321$. The tilt angle is negative for days ($n=50$–130) in Zahedan city which means the face of collector is in north direction during these days. The two time (20 Mar to 21 Sep, 22 Sep to 20 Mar) adjustment of tilt angle increase the energy gain by 8%.
Agha and Sbita [89]	The optimum tilt angle is selected that gives best matching load curve with solar irradiation curve and is given by following optimization factor ($F_{opt}(\beta)$), in terms of monthly average daily total solar irradiation ($H_{avg}(\beta)$), curve standard deviation ($SDEV(\beta)$). $F_{opt}(\beta)=H_{avg}(\beta)/SDEV(\beta)$
Talebizadeh et al. [90]	The seasonal optimum slope angle developed by correlation is $\beta_{opt}=1.073\phi+10.3$ in winter season, $\beta_{opt}=0.4885\phi-10.27$ in spring season, $\beta_{opt}=0.2637\phi+4.961$ in summer season, $\beta_{opt}=0.8966\phi+23.81$ in autumn season and $\beta_{opt}=0.6804\phi+7.203$ is yearly optimum tilt angle. The optimum daily, monthly, seasonally and yearly azimuth angle is zero for Iran. The energy gain is maximum at daily optimum tilt angle and solar tracking system is not economical due to high cost.

one year and emphasized the need for validating the analytically determined optimal tilt angles experimentally also. Li et al. [72] proposed a sunshine hour data based model to calculate solar radiation on the tilted surface with different orientations. The maximum error in this model is found to be less than 5.2% so it can be used to determine the optimum tilt angle for sites where only sunshine hour data are measured. Chang [73] used orthogonal array experiment technique and an ant direction hybrid differential evolution algorithm (ADHDEOA) for tilt angle of PV modules in Taipei area. An ant colony method is used to find the optimum tilt angle at which power is maximum. The annual optimal angle for Taipei area is determined (Taichung 17.3°, Tainan 16.15°, Kaosiung 15.79°, Hengchung 15.17°, Hualian 17.16°, Taitung 15.94°). The computer simulated optimum angles are also verified experimentally. The ADHDEOA is found to be better than GA method in computation and is much effective in optimum tilt angle determination.

Beringer et al. [74] carried out the experimental measurements for monthly optimum tilt angles for cloudy mid-latitude location Hannover, Germany, using eight multi-crystalline silicium solar photovoltaic panels with south orientation mounted at different tilt angles from 0° to 70° in steps of 10°. The maximum power is found to be at 50–70° tilt angles in winter and 0–30° in summer months. The yearly performance difference is found to be less than 6% showing minor differences at various tilt angles. However, further follow-up studies to determine optimal tilt angles for such locations with cloudy conditions can further be taken up using the optimization techniques. Pour et al. [75] used isotropic Liu and Jordan model [12] to determine the optimum tilt angle and the non-isotropic Klein [23] and Hay [13] models, which consider the effects of the azimuth angle, are used to determine the optimum tilt and azimuth angles for Isfahan, Iran [lat 32° N, Long 51° E]. The optimum tilt angles determined using the Liu and Jordan model for south orientation shows that the yearly optimum tilt angle is nearly equal to the latitude of the site, whereas the results using the non-isotropic models i.e. the Klein and Hay models show that the panels at optimum tilt and azimuth angles receive higher solar energy compared with the fixed solar panel which indicates that the determination of both optimum tilt and azimuth angles is important for maximizing incident solar radiation at a location.

A number of studies have been carried out to maximize power output and cooling load reduction of building integrated photovoltaic [BIPV] as such the optimum tilt angle determination for BIPV design and development is another important area of PV applications. Elhassan et al. [76] determined optimum tilt angles for building integrated photovoltaic design and applications at Kuala Lumpur, Malaysia [Lat 2° 30′ N Long 112° 30′ E], by using a four PV module experimental setup inclined in the North, South, East and West directions to determine the optimum tilt angles for these directions as solar radiation received during the day varies for each direction. The optimum tilt angle for this location was found to be nearly equal to the latitude of the location. Sunderan et al. [77] investigated that the optimum tilt angle and orientation enhance the power generation of Standalone Photovoltaic Electricity Generation Systems (SPVEGS) in Ipoh, Malaysia. The orientation of PV module is North from April to August and south for the other months. The energy gains of 6.4% and 6.1% are achieved for monthly and yearly optimum tilt angles respectively. Lucio et al. [78] have given an algorithm to evaluate optimum tilt angle for obtaining minimum loss of load probability and optimum design of stand-alone photovoltaic systems in Europe. Asowata et al. [79] used a photovoltaic experimental setup at Vaal Triangle, South Africa, lying in the southern hemisphere [Lat. 29° 00′ S and Long 24° 00′ E] to validate the optimum tilt angles 16°, 26° and 36 ° determined analytically for winters for this location. Sun et al. [80] determined the optimum tilt angles for the shading-type BIPV

Table 4
Optimum tilt angles for solar panels for different locations.

Study	Location	Monthly optimum tilt angle	Yearly optimum tilt angle	Remarks
Tsalides and Thanailakis [98]	Greece		$\pm 60^\circ$ (Greater than latitude)	– Optimum tilt angle decreases with large azimuth angles. – Solar radiation uniformity is considered for yearly optimum tilt angle calculation resulting in continuous supply of power to load.
Benghanem [99]	Madinah, Saudi Arabia	Winter 37° Summer 12°	23.5° equal to latitude	– At yearly optimum tilt angle, the energy loss is 8% in comparison to monthly optimum tilt angle. – For more accurate tilt angle anisotropic models, optimization techniques need to be considered.
Rowlands et al. [100]	Ontario Canada		Equal to latitude	– Azimuth angle is close to south. – PV module power in outdoor conditions should be investigated.
Kaldellis and Zafirakis [101]	Athens	15° with ($\pm 25^\circ$) deviation for summer		– Performance difference of PV at fixed and variable optimum tilt angle is 27.3%.
Calabròa [102]	USA and Europe		$\beta_{opt}=\phi-(26^\circ, 27^\circ, 28^\circ)$ where ϕ varies from 36° to 46°	– Semi-fixed solar panels are beneficial due to different seasonal angles.
Agarwal et al. [103]	Delhi and Nandha in Haryana	Tilt angle is minimum in summer and maximum in winter season.	Calculated angle by Liu and Jorden, Reindel, Hay and Badescu models is 28.29°, 29.41°, 29.08°, 27.58° for Delhi and 30.61°, 31.66°, 31.20°, 29.58° for Nandha.	– Incident solar radiation is maximum at monthly tilt angles and minimum at yearly optimum tilt angle.
Zhao et al. [104]	Singapore	Daily, monthly average varies from 0.1° to 25.9°, 0.1° to 28.7°.		– Energy (kWh/m^2-year) received at 0°, monthly, daily tilt are 1264.78, 1297.32, 1298.86 proving monthly tilt angle change, is beneficial.
Daut et al. [105]	Perlis, Northern Malaysia	−17.160° to 29.740° ($\pm$ means PV facing is north, south)		– $\beta_{opt}=\phi-\delta$. – Changing PV orientation and tilt is necessary. – Monthly beam solar irradiance (968.36 W/m^2) can be increased at optimum tilt angle so good scope of PV generation.
Khatib et al. [106]	Kuala Lumpur, Johor Bharu, Ipoh, Kuching , Alor Setar in Malaysia	Maximum in summer and minimum in winter		– Monthly optimum tilt angle is recommended.
Kacira et al. [107]	Sanliurfa, Turkey	13° and 61° for June and December months respectively.		– Solar radiation gain at monthly optimum tilts angles is more than seasonal and yearly tilt angles. In two-axis solar tracking system, there is a gain of 29.3% in solar radiation resulting in 34.6% gain in generated power. – Optimization techniques can be used for better accuracy.
Yakup and Malik [108]	Brunei Darussalam	Sep 1.6°, Dec 32.3°	33°	– Monthly tilt angle is recommended. – Correlation of solar noon optimum tilt angle in terms of latitude are given, which are useful to collect maximum radiation in winter
Ahmad and Tiwari [109]	New Delhi	Winter 47.5° ($\phi+90^\circ$), summer 13° ($\phi-60^\circ$)	Equal to the latitude	– The loss of energy is 1 % and 15 % when tilt angle is adjusted seasonal and yearly instead of monthly optimum tilt angle. – Using measured diffuse radiation in Anisotropic models can improve the results
Gunerhan and Hepbasli [110]	Izmir, Turkey	Summer $\beta_{opt}=\phi+15^\circ$ winter $\beta_{opt}=\phi-15^\circ$	Equal to latitude	– Extraterrestrial radiation is used for tit angle calculation resulting in inaccuracy.
Ulgen [111]	Izmir, Turkey	June 0°, December 61°	30.30°	– Total solar radiation at 0°, 61°, 30.30° tilt angles are 27.07 MJ/m^2/day, 10.81 MJ/m^2/day and 6397.78 MJ/m^2/year.
Elhab et al. [112]	Kuala Lumpur, Malaysia.		$\beta=10^\circ$	– Cooper's equation, Visual Basic Application programming are used
Xianping [113]	Changsha		19.22° i.e. 10° less than latitude.	– Anisotropic Hay's model, ρ of 0.14 (Magallanite) is used to calculate optimum tilt angle.

claddings at different orientations for Hong Kong [lat 22.25°N Long 114.1667°S]. The maximum electricity generation per unit PV area is found when the PV modules are installed on south façades at the tilt angle of 10°, thus increasing the energy gain significantly.

Bojić et al. [81] determined optimum tilt angles for PV systems that are located in four towns (Les Avirons, Petite-France, Saint-Benoit,and Piton Saint-Leu) in Reunion Island, France. The Hooke–Jeeves algorithm is used to find optimum tilt angle at which electrical energy generation becomes maximum and found a discrepancy between latitude and optimum tilt angles, showing the need for determining optimum tilt angles for every location. Siraki and Pillay [82] proposed a modified HDKR (Hay, Davies, Klucher, Reindl model) anisotropic sky model which also considers the effects of the adjacent buildings in urban locations to calculate the optimum angle. The optimum angles for the five different latitudes were determined and the tilt angle dependence on latitude, weather conditions, surrounding obstacles and optimum azimuth was found. The optimum tilt angle is found to be close to the location latitude for small latitude, while for higher latitude the optimum angle is found to be smaller than the location latitude. These studies indicate that for maximum solar radiation capturing both orientation and optimum tilt angle of PV array needs to be determined accurately for any location.

Several authors have given correlations of optimum tilt angle (β_{opt}) in terms of different parameters for the calculation of optimum tilt angles, as shown in Table 3.

Hartely et al. [91] compared Temps–Coulson's, Klucher's and Hay's models for the estimation of total radiation on vertical planes facing north, south, east and west to find the best model that can be used to calculate solar irradiance on tilted planes in Valencia Spain [Lat 39.5° N, Long 0.67° W]. Hay's model was found to be accurate in determining hourly variations of optimum tilt angles for south facing collector but the average monthly optimum tilt angle was not given accurately. Tang and Wu [92] developed a method to estimate the optimal tilt angles of a solar collector based on the monthly horizontal radiation and prepared a map showing optimal tilt angles for the year for south facing collectors in China and used Collares-Perara and Rabl [93] correlation to estimate diffuse solar radiation and Klein's method [23] for the calculation of Rb. It is found that Collares-Perara and Rabl correlations give accurate optimum tilt angle except for low clearness index sites.

Fahl and Ganapathisubbu [94] computed optimum tilt angles for Bangalore [Lat 12.97° N, Long 77.56° E] under various tracking conditions. The optimum tilt angles are estimated between 15° and 17° for south facing collectors. The incident solar radiation is found to be the same within variation of $\pm 3°$ in optimum tilt angle. The output of a monthly adjusted collector at optimum angle is found to be increased by 10% in comparison to that received at horizontal orientation. But the output is found to be 35% more for collectors with tracking system.

Chang [95] calculated the optimal tilt angles for six locations in Taiwan using the extraterrestrial, predicted global radiation model and ten-year measured solar radiation data. The results indicate that the tilt angles calculated from extraterrestrial, predicted solar radiation are latitude dependent whereas those calculated using measured data vary from site to site and shows that the optimal tilt angle is accurately determined using measured data for a location with cloudy or pollutant environment. The method of maximizing extraterrestrial radiation for tilt angle calculation does not consider attenuation of the solar radiation so it will not give accurate tilt angles. Shariah et al. [96] used the annual solar fraction as an indicator to find the optimum tilt angles for a thermosyphoning solar water heater in the northern and southern parts of Jordan. The optimum tilt angle varies from ϕ to $\phi \pm 20°$ for high solar fraction system. Tang et al. [97] developed a mathematical procedure for determining optimum tilt angles based on maximum annual collectible radiation for evacuated glass tube collectors in China and found that the optimum tilt angles are found to be about 10° less than the latitude for sites with latitudes greater than 30°.

The optimum tilt angle study highlights for different locations by the other authors are summarized in Table 4.

5. Conclusions

In this paper, optimum solar panel tilt angle methods using different optimization techniques are reviewed. Based on the study, the main conclusions are as follows:

- Optimum tilt angle determination maximizes the incident solar radiation, resulting in optimum sizing of solar systems.
- The optimum tilt angles are determined by changing tilt angle from 0° to 90° in different steps and using GA, SA, PSO and ANN optimization techniques.
- The latitude based thumb rules mostly used by PV and solar thermal industry may be suitable for certain locations but result in escalating the system costs and oversizing of systems if used without proper analysis. This effect will be more prominent in case of solar power plants where a large number of solar panels are used.
- The energy gain is found to be more at monthly optimal tilt angle than seasonal and yearly tilt angles for any location.
- The ANN models can further be investigated using different combinations of input variables such as horizontal solar radiation, latitude, extraterrestrial radiation, the isotropic and anisotropic models, diffuse and beam radiation for estimating optimum tilt angle.
- For better accuracy of optimum tilt angle calculations different anisotropic models and optimization techniques can be tested at different sites.
- For urban areas, the obstacles affecting the solar radiation should also be considered for computing optimum tilt angles.

The results of different optimization and ANN techniques can be compared to identify the best methods for further research. Although, a number of studies have been carried out using predicted solar radiation data, extraterrestrial radiation data but the determination of these angles using measured global, diffuse and beam solar radiation values will result in better accuracy. However, this may require elaborate reliable solar radiation measuring equipments at the location. It will be interesting to study the effect of tilt angles on annual energy production and economics of solar power plants installed at various locations worldwide. This will provide valuable inputs to researcher and solar industry to install efficient and cost effective solar systems at different sites using optimal tilt angles for each location worldwide.

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